Application of Biomechanical Methods to Cardiac Simulation

Culture Sciences de l'Ingénieur

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Published in :
April 2025

école normale supérieure paris–saclay

This document stems from a one-year internship at the Institute of Computer-Assisted Cardiovascular Medicine at the German Heart Center of the Charité (Berlin, Germany), under the supervision of Mr. Obermeier in the research group led by Mr. Goubergritz. The author is in the final year of study at the École Normale Supérieure Paris-Saclay. After completing a Master 1 in mechanical engineering, he specialized in fluid mechanics during his Master 2 at the Institut Polytechnique de Paris.

The aim of this article is to propose an application of biomechanics, as presented in the Culture Sciences de l'Ingénieur articles by Thomas LAVIGNE and Bastien SAUTY [10] [9].

1 Introduction

The cardiovascular system is a complex network of vessels and chambers that enable the circulation of blood throughout the body. At the centre of this system lies the heart, a hollow muscle composing of four cavities responsible for pumping blood to deliver oxygen and nutrients to tissues to all body parts while removing waste products. The intricate mechanics of the heart involve the coordinated function of valves, chambers, and blood vessels, which together ensure efficient circulation.

Understanding the dynamics of the cardiovascular system is crucial for diagnosing and treating cardiovascular diseases, which are among the leading causes of mortality worldwide. Advances in computational modelling have enabled researchers to simulate the behaviour of the heart and its components with increasing accuracy. This article explores the modelling of heart valve in the left heart as important structure to ensure cardiac function, using fluid-structure interaction (FSI) techniques. By integrating both fluid dynamics and solid mechanics, these models provide valuable insights into the hemodynamics of the heart, aiming to facilitate the development of personalized treatment strategies and improved clinical outcomes.

2 The cardiovascular system

The cardiovascular system consists of the heart, blood vessels (arteries, veins, and capillaries) and blood. It is responsible for the transport of blood from the heart to the extremities and various organs, and then back from these organs to the heart. This system plays a critical role in supplying oxygen and nutrients to tissues while removing waste products.

2.1 The heart

The heart is is a muscular organ that functions as a pump. It is divided into two part, the right heart (in blue on Fig. 1) and the left heart (in red on Fig. 1). The two heart sides have a similar anatomies: a receiving chamber, called atrium (right or left), and a discharging chamber, called ventricle (right or left).

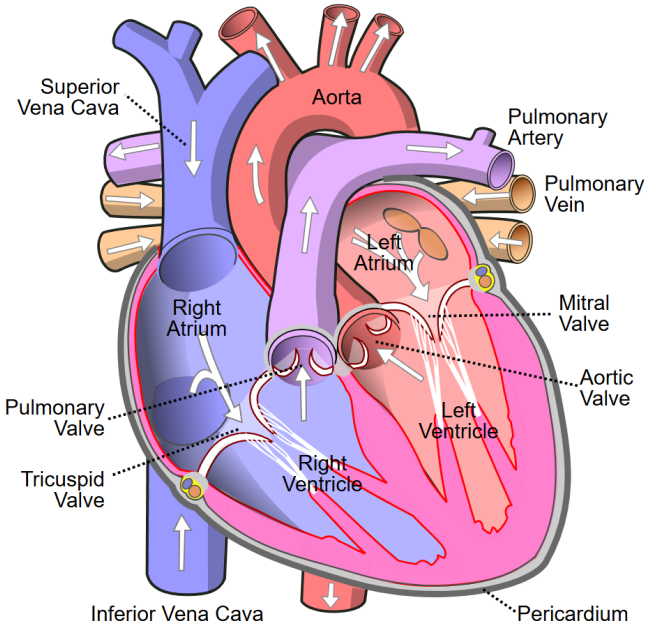


Figure 1: Heart diagram

During a cardiac cycle, the atrium dilates (atrial diastole) to fill with blood and then, by contracting (atrial systole), ejects the blood into the ventricle, which dilates simultaneously (ventricular diastole). The contraction of the ventricle (ventricular systole) then ejects the blood from the ventricle into the aorta (left heart) or the pulmonary artery (right heart). The heart valves direct the blood flow from the atria to the ventricles and from the ventricles to the arteries.

2.2 The vascular system

The circulatory system is divided into two major circuits: the systemic circulation and the pulmonary circulation (Fig. 2).

The pulmonary circulation carries deoxygenated blood from the right ventricle via the pulmonary artery to the lungs, where it is oxygenated. Oxygenated blood is then returned to the left atrium of the heart through the pulmonary vein, ready to be pumped into the left ventricle the systemic circulation.

The systemic circulation, on the other hand, delivers oxygenated blood from the left atrium of the heart, through the aorta, to the rest of the body. Then, deoxygenated blood is collected and returned to the right side of the heart via the inferior and superior vena cava, completing the cycle.

Due to its function, the left ventricle, which supplies blood to almost the entire body, is subjected to a greater pressure, leading to higher stresses in left heart valves (aortic and mitral). Thus, these valves are more prone to dysfunction and diseases that are clinically relevant and can lead serious, life-threatening, patient states.

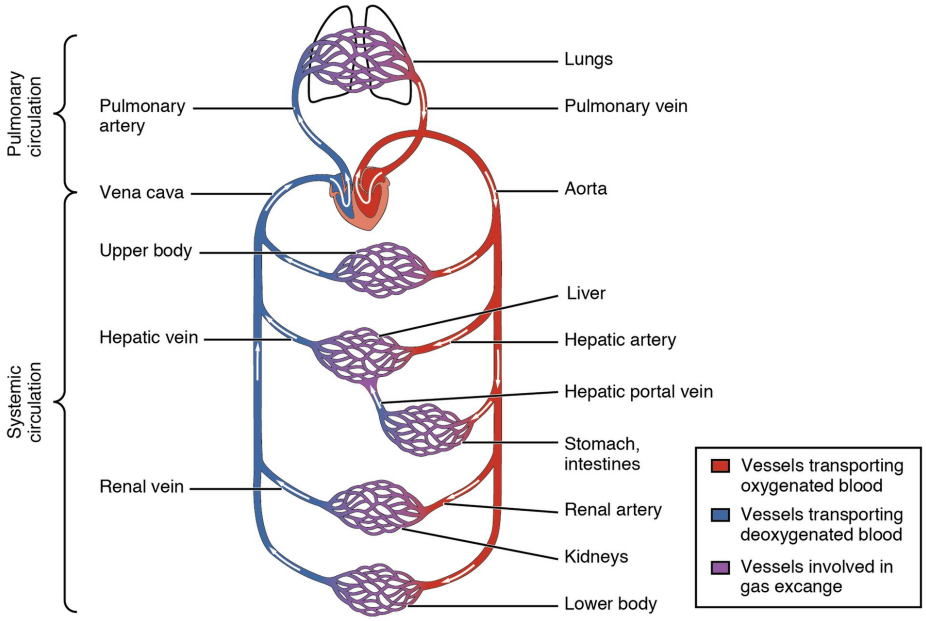


Figure 2: Diagram of the cardiovascular system

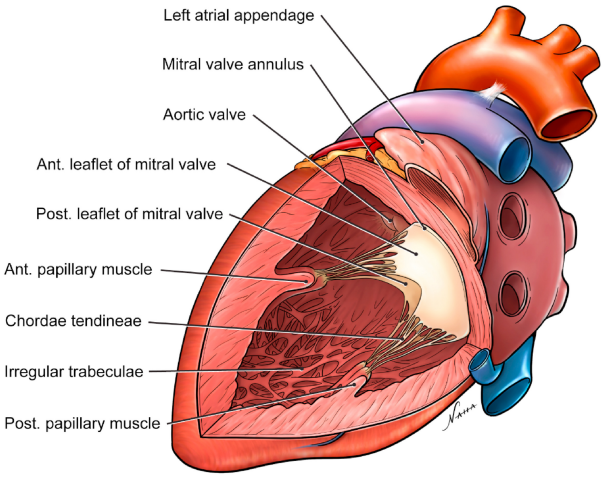
2.3 Focus on the left heart valves

The mitral and aortic valves have similar functions, but exhibit significant anatomical differences.

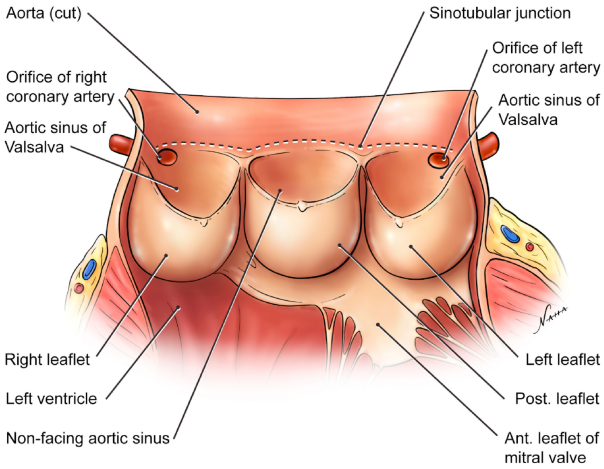
Both are passive systems that react to blood flow. During the systole of the upstream chamber, the increase in pressure allows the leaflets to open, letting blood flow through. Then, during the diastole of the upstream chamber, the decrease in pressure causes the leaflets to close, preventing blood reflux.

- The mitral valve apparatus (Fig. 3a) has a complex anatomy, consisting of the annulus, two leaflets (anterior and posterior one) and the valvular subsystem composed of the chords and the papillary muscles. A chord like structure emerges from the tip of either of the two papillary muscles (PMs) and inserts into the leaflets. The two PMs as known as anterolateral and posteromedial, and are finger-like projections from the wall of the left ventricle. These chords avoid the MV to prolapse into the left atrium and ensure the proper closure of the valve.The surface area of a healthy MV in an adult ranges from 4 to 6 cm^2.
- The aortic valve apparatus (Fig. 3b) consisting of the annulus, three leaflets (right coronary, left coronary, and non-coronary), and the supporting structures within the aortic root. The leaflets are semi-lunar structures that attach to the annulus and meet at the commissures, forming a coaptation area that ensures valve competency. The aortic root, which houses the valve, includes the sinuses of Valsalva, which are dilations behind each leaflet that facilitate valve opening and closure. The sinotubular junction, where the aortic root meets the ascending aorta, provides additional structural support. The surface area of a healthy aortic valve in an adult typically ranges from 3 to 4 cm^2.

3 Modelling of physical continua



(a) Lateral view of the heart with a portion of the left ventricular wall removed, revealing the mitral valve, chords, and papillary muscles [6]



(b) Isolated view of the aorta cut open to show aortic valve's leaflets and sinuses [6]

Figure 3: Anatomy of the mitral and aortic valves and their integration in the cardiac structure

The heart's function is inherently a multi-physics system, encompassing complex interactions between fluid dynamics and solid mechanics. To effectively study and simulate these interactions in silico, it is essential to model these physical aspects numerically. This section introduces the modeling of the physical continuum, laying the groundwork for understanding how solid and fluid dynamics, along with fluid-structure interaction (FSI), are integrated to capture the heart's intricate behavior. By bridging medical observations with numerical simulations, we aim to provide a comprehensive framework for advancing cardiac research and clinical applications.

3.1 Solid mechanics modelling

Cardiac valve biological material is commonly modelled using hyperelastic models. Hyperelastic models aim to reproduce the behaviour of nonlinear materials under large strain loading.

During the cardiac cycle, the mitral valve undergoes significant strains when it opens and closes. Sacks et al. [8] demonstrate a deformation of up to 75% in certain areas of the anterior leaflet of a porcine mitral valve during *in vitro* experiments of opening and closing cycle. These deformations are too complex for linear elastic models to capture. Hyperelastic models can describe these large deformations through strain-energy functions, providing a more realistic representation of the valve's material properties.

The mitral valve's tissues exhibit complex mechanical behaviours, including anisotropy, which refers to the direction-dependent properties of the material. This anisotropic behaviour is largely influenced by the arrangement of elastin and collagen fibers (Fig. 4). Additionally, these tissues display nonlinear behaviors that are effectively captured by hyperelastic models. These models also allow for accurate simulation of the valve's interaction with the blood flow that drives opening and closing of the valves, which can enhance for understanding conditions like mitral regurgitation or prolapse.

The hyperelastic models used for the modelling of biological tissues are presented in the Culture Sciences de l'Ingénieur article by Thomas LAVIGNE and Bastien SAUTY [9]. The subsequent

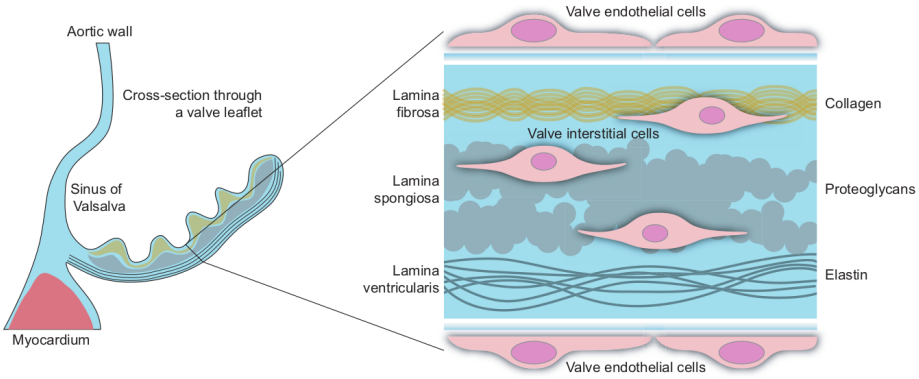


Figure 4: Simplified structure of the human aortic valve leaflet. On the left is a schematic cross section through a leaflet of the aortic valve [11]

simulations are based on the same models.

3.2 Fluid dynamics modelling

Newtonian fluids, such as water, have a constant viscosity μ, in contrast to non-Newtonian fluids whose viscosity depends on the shear rate $\dot{\gamma}$. Blood volume is composed of approximately 55 % of blood plasma, which mainly consists of water and can be considered as Newtonian [2]. However, the remaining part of the blood is composed partly of cells, primarily red blood cells, which exhibit a non-Newtonian behaviour. At high shear rates, blood cells deform and facilitate the circulation of blood flow, causing the viscosity of the fluid to decrease. At low shear rates, the red blood cells aggregate and form a phenomenon called Rouleaux, which leads to an increase in viscosity [4]. In other words blood exhibits a shear thinning behaviour.

This non-Newtonian behaviour of blood is commonly modelled using the Carreau-Yasuda model [3] [12]:

$$\mu(\dot{\gamma}) = \mu_\infty + (\mu_0 - \mu_\infty)(1 + (\lambda\dot{\gamma})^a)^{(n-1)/a} \tag{1}$$

where μ_∞ is the viscosity at infinite shear rate, μ_0 the viscosity at zero shear rate, λ the relaxation time, a a shear-thinning parameter, and n the power constant.

The governing equations for a fluid flow are the Navier-Stokes continuity and momentum equations. The continuity equation is expressed as:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.(\rho \mathbf{v}) = 0 \tag{2}$$

where $\mathbf{v}$ is the velocity vector field and ρ the fluid density. $\boldsymbol{\nabla}$ denotes the mathematical nabla operator. For an incompressible flow, the density ρ is constant. Thus, the transient term in Eq. 2 is zero and it simplifies to:

$$\boldsymbol{\nabla}.\mathbf{v} = 0 \tag{3}$$

The Navier-Stokes momentum equation can be reguarded as being a formulation of the conservation of mommentum, and is expressed as:

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \boldsymbol{\nabla}.(\rho \mathbf{v} \otimes \mathbf{v}) = \boldsymbol{\nabla}.\boldsymbol{\sigma} + \mathbf{f}_b \tag{4}$$

where $\otimes$ denotes the outer product, $\boldsymbol{\sigma}$ is the stress tensor and $\mathbf{f}_b$ is the body force per unit volume caused by external forces (such as gravity). The stress tensor can be decomposed into a volumetric and deviatoric term. The volumetric stress is caused by the pressure p and the deviatoric term is due to viscosity:

$$\boldsymbol{\sigma} = -p\mathbf{I} + \boldsymbol{\tau} \tag{5}$$

where $\boldsymbol{\tau}$ is the viscous stress tensor an $\mathbf{I}$ is the identity matrix. The viscous stress tensor can be expressd as a function of the viscosity μ and the strain tensor $\boldsymbol{\epsilon}$:

$$\boldsymbol{\tau}(\dot{\gamma}) = 2\mu(\dot{\gamma})\boldsymbol{\epsilon} \tag{6}$$

where the strain tensor, for infinitesimal deformations, is given by:

$$\boldsymbol{\epsilon} = \frac{1}{2}\left(\boldsymbol{\nabla}\mathbf{v} + (\boldsymbol{\nabla}\mathbf{v})^T\right) \tag{7}$$

It should be noted that in the case of a Newtonian fluid, the shear stress tensor is only a function of the strain tensor.

Thus one can compute the shear rate from the strain tensor:

$$\dot{\gamma} = \sqrt{2\boldsymbol{\epsilon} : \boldsymbol{\epsilon}} \tag{8}$$

Finally injecting Eq. 8 and 6 in Navier-Stokes equation 4 leads to:

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \boldsymbol{\nabla}.(\rho \mathbf{v} \otimes \mathbf{v}) = -\boldsymbol{\nabla}.(p\mathbf{I}) + \mu(\dot{\gamma})\boldsymbol{\nabla}^2\mathbf{v} + \mathbf{f}_b \tag{9}$$

In Eq.9, the first term on the left-hand side captures the transient behaviour of the flow, describing how velocity changes over time. The second term for convection, representing the tendency of a particle to move with the bulk velocity of its surrounding particles.

On the right-hand side, the first term indicates that fluids move towards the direction of the steepest (negative) pressure gradient. The second term is a diffusion term, arising from the fluid's viscosity. The third term on the RHS, as previously mentioned, represents the influence of body forces like gravity on the flow.

It is important to note that the Navier-Stokes momentum equation, Eq. 9, involves vector fields and thus constitutes separate equations for the x, y, and z directions.

3.3 Fluid Structure Interaction modelling

Numerical simulation of problems involving the interaction between a fluid and a solid continuum can be conducted using two main techniques: one-way or two-way coupled Fluid-Structure Interaction (FSI) [1].

In **one-way FSI**, only the fluid exerts an influence on the structure, while the deflection of the structure does not affect the fluid flow (Fig. 5a). This approach is typically employed when

the structure is relatively stiff or the fluid forces are weak, such that the structure does not significantly alter the flow characteristics. The governing equations for the fluid domain are solved independently, and the resulting fluid forces are applied as external loads to the structural solver.

The primary advantage of one-way FSI is its computational efficiency, as it avoids the need for iterative coupling between the fluid and structural solvers. This method is suitable for scenarios where the structural deformation is minimal or negligible, such as in the case of a rigid or nearly rigid structure interacting with a fluid flow.

Two-way FSI, on the other hand, involves a more complex interaction where both the fluid and the solid structure influence each other reciprocally (Fig. 5b). In this approach, the fluid flow exerts a loading on the structure, which, by deforming, affects the flow. This reciprocal interaction is governed by the equations of motion for both the fluid and the structure, coupled through boundary conditions at the fluid-structure interface.

At the fluid-structure interface, the kinematic and dynamic continuity conditions must be satisfied:

$$\mathbf{v}_f = \mathbf{v}_s, \tag{10}$$

$$\sigma_f \cdot \mathbf{n}_f = \sigma_s \cdot \mathbf{n}_s, \tag{11}$$

where $\mathbf{v}_s$ is the structural velocity, and $\mathbf{n}_f$ and $\mathbf{n}_s$ are the normal vectors at the interface for the fluid and structure, respectively.

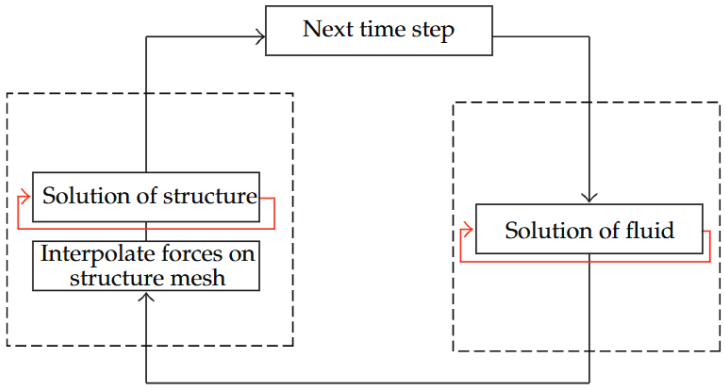
Two-way FSI is particularly suited for simulations involving significant structural deformation that modifies the surrounding flow, such as the interaction between a flexible valve and blood flow. However, this method implies high computational costs, as it requires an iterative convergence algorithm to compute the position of the structure and the fluid flow at each time step.

4 Application in cardiac Modelling

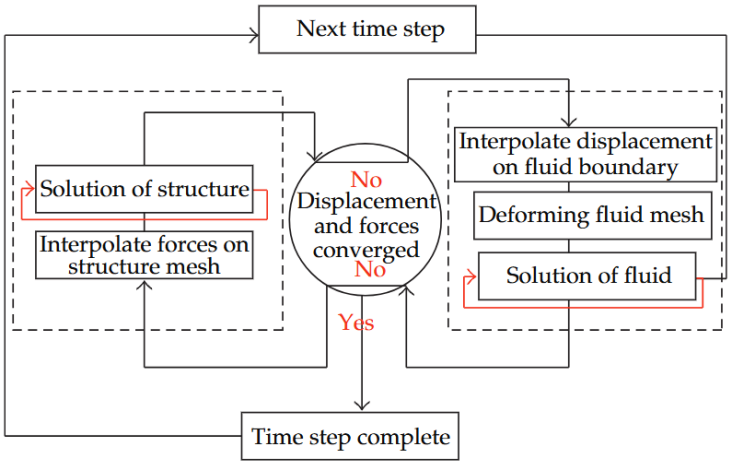
4.1 Modelling the left heart

The simulation of the left heart [7] allows for patient-specific numerical reconstruction of cardiac structures, providing valuable insights into intra cardiac hemodynamics. This process involves meticulously reconstructing anatomical features, including the left ventricle, left atrium, aorta, mitral valve, and aortic valve, using high-resolution MRI sequences (Magnetic Resonance Imaging) or CT-scan (Computed Tomography). The study presents two distinct methodologies for modelling the left heart, each tailored to different clinical and research needs.

Rigid Wall Model (RWM): This simplified approach computes systole and diastole separately, bypassing the need to model the complex valve opening and closing processes. Instead, it uses a velocity outlet boundary condition to simulate ventricular movement. The RWM approach assumes a rigid ventricular wall, which simplifies the computational process and reduces data requirements and computation times. This makes RWM particularly suitable for clinical applications where rapid results are essential, such as predicting diastolic inflow after mitral valve treatment or stress testing.



(a) One-way coupling



(b) Two-way coupling

Figure 5: Solution algorithms for one-way and two-way coupling [1]

Prescribed-Motion Fluid-Structure-Interaction (PM-FSI): This approach integrates ventricular movement over the entire cardiac cycle, providing a comprehensive simulation of intracardiac hemodynamics. It models the opening and closing processes of the cardiac valves using Darcy's law, which describes the pressure drop across a porous medium. The pressure drop is calculated as:

$$\Delta p = \frac{\mu \cdot L \cdot Q}{k \cdot A} \tag{12}$$

where μ is the dynamic viscosity, L is the valve thickness, Q is the flow rate, k is the permeability, and A is the valve area. The opening and closing of the valves are controlled by modifying the permeability k. This method is particularly suitable as it allows maintaining a continuous mesh at the valve level while reproducing a good separation of the different cardiac cavities.

This approach enables the computation of cyclic flow patterns, blood washout, energetic characteristics, and deformation-related phenomena, making it suitable for detailed hemodynamic analysis.

Both models where compared to 4D flow MRI measurements, which is the state of the art method to measure blood flow inside the heart in a living organ. The results of the left heart simulations demonstrate a strong agreement with *in vivo* measures, particularly in capturing key

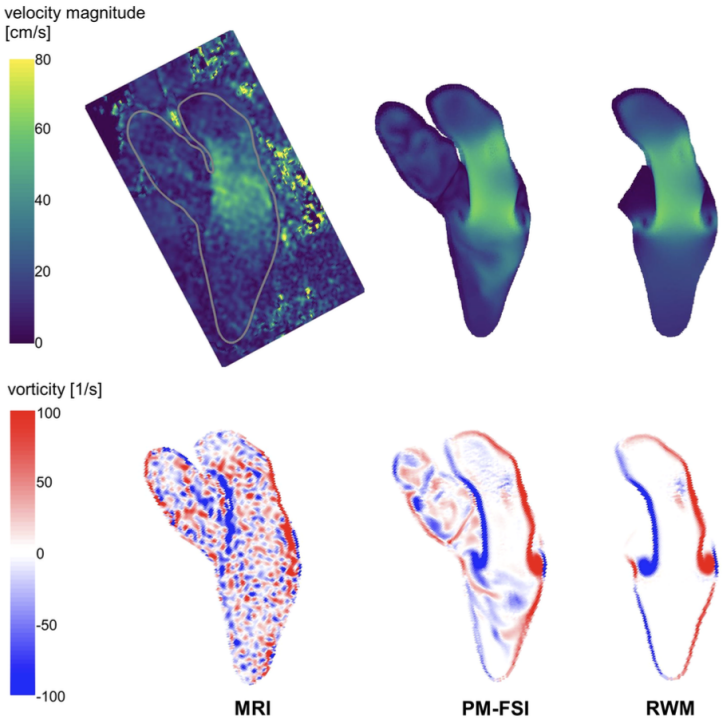


Figure 6: Plane cut through the left ventricle, showing the velocity magnitude at the early phase of ventricular diastole, as well as the out-of-plane vorticity at peak ventricular diastole (bottom) for 4D flow MRI (left), PM-FSI (mid), and RWM (right) [7]

hemodynamic parameters such as flow rates, valve velocities, and kinetic energy. As illustrated in Fig. 6, the simulated flow patterns closely resemble those observed in vivo, validating the accuracy of the models.

These simulations provide valuable insights into intra-cardiac blood flow dynamics, offering a non-invasive approach to assess cardiac function and diagnose potential abnormalities. The utility of such simulations extends to clinical applications, where they could aid in treatment planning, predicting outcomes of surgical interventions, and enhancing our understanding of cardiovascular diseases.

4.2 Advanced Simulation of Heart Valves

The simulation of heart valves has evolved significantly, transitioning from simplified models that use Darcy's law to simulate valve dynamics to more sophisticated fluid-structure interaction (FSI) approaches. While simplified models provide valuable insights into basic valve behavior, they often fall short in capturing the complex interactions between blood flow and valve mechanics.

In contrast, the full FSI approach, as detailed by Le et al. (2022), offers a comprehensive framework for simulating heart valves within patient-specific left heart anatomies [5]. This method integrates both fluid and structural solvers, allowing for a detailed representation of the dynamic behavior of valves throughout the cardiac cycle. By coupling these solvers, the FSI approach can simulate the intricate interactions between blood flow and valve leaflets, providing a more accurate depiction of valve dynamics.

Clinical Implications and Patient Data

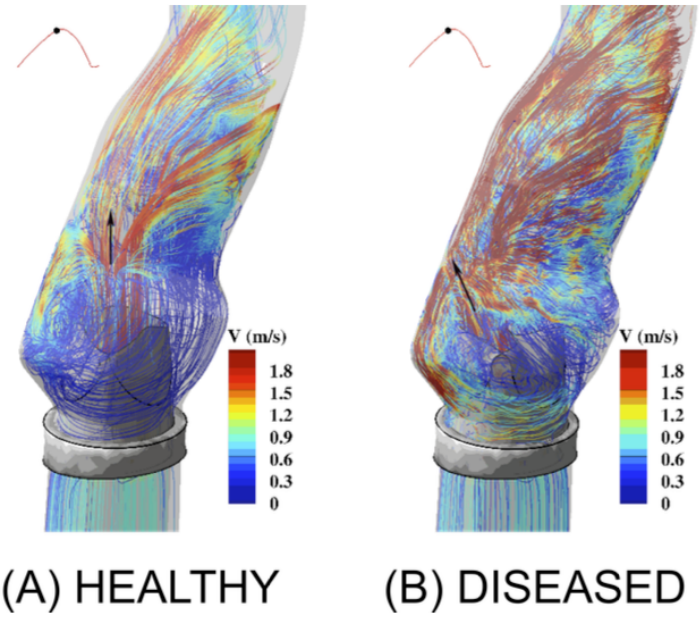


Figure 7: Comparison of flow patterns under healthy and diseased conditions, highlighting the differences in flow dynamics that can be analysed through advanced FSI simulations [5]

The advanced FSI simulations provide valuable insights into the hemodynamics of the left heart, as illustrated in Fig. 7. These simulations allow access to detailed flow patterns, such as vorticity and velocity fields, which are essential for assessing valve function and identifying potential issues.

For patients, such simulations offer the potential for personalized treatment planning. By incorporating patient-specific anatomical data, these models can predict the outcomes of surgical interventions, optimize valve designs, and improve our understanding of cardiovascular diseases. The ability to visualize and quantify flow characteristics, as shown in Figure 7, enables clinicians to make more informed decisions, ultimately enhancing patient care and outcomes.

5 Conclusion

The simulation of the cardiovascular system, particularly the left heart and its valves, has significantly advanced with the application of fluid-structure interaction techniques. These models provide a detailed representation of the dynamic interactions between blood flow and cardiac structures, offering valuable insights into normal and pathological conditions.

The ability to simulate patient-specific scenarios allows for personalized treatment planning, enabling clinicians to predict the outcomes of surgical interventions and optimize therapeutic strategies. As computational methods continue to evolve, the integration of advanced modeling techniques with clinical data promises to enhance our understanding of cardiovascular diseases and improve patient care.

By bridging the gap between theoretical modeling and clinical application, these simulations pave the way for more effective and personalized treatment options, ultimately contributing to

better health outcomes for patients with cardiovascular diseases.

References

[1] F.-K. Benra et al. “A Comparison of One-Way and Two-Way Coupling Methods for Numerical Analysis of Fluid-Structure Interactions”. In: *Journal of Applied Mathematics* (2011). DOI: 10.1155/2011/853560.

[2] M. Brust et al. “Rheology of Human Blood Plasma: Viscoelastic Versus Newtonian Behavior”. In: *Physical Review Letter* (2013). DOI: 10.1103/PhysRevLett.110.078305.

[3] P. J. Carreau. “Rheological Equations from Molecular Network Theories”. In: *Journal of rheology* (1972). DOI: 10.1122/1.549276.

[4] G. Carty, S. Chatpun, and D. M. Espino. “Modeling Blood Flow Through Intracranial Aneurysms: A Comparison of Newtonian and Non-Newtonian Viscosity”. In: *Journal of Medical and Biological Engineering* (2016). DOI: 10.1007/s40846-016-0142-z.

[5] Trung Bao Le et al. “Computational Methods for Fluid-Structure Interaction Simulation of Heart Valves in Patient-Specific Left Heart Anatomies”. In: *Fluids* 7.3 (2022), p. 94.

[6] Shinelle Whiteman Yusuf Alimi Mark Carrasco Jerzy Gielecki Anna Zurada Marios Loukas. “Anatomy of the cardiac chambers: A review of the left ventricle”. In: *Translational Research in Anatomy* (2020). DOI: 10.1016/j.tria.2020.100095.

[7] L. Oberneier et al. “Verification Study of In Silico Computed Intracardiac Blood Flow With 4D Flow MRI”. In: *IEEE Transactions on Biomedical Engineering* (2024). DOI: 10.1109/TBME.2024.3381212.

[8] M. S. Sacks et al. *Surface Strains in the Anterior Leaflet of the Functioning Mitral Valve*. Vol. 30. The Biomedical Engineering Society, 2002.

[9] SAUTY Bastien LAVIGNE Thomas. *Caractérisation mécanique d’un tissu mou : le muscle passif - Culture-Sciences de l’Ingénieur*. https://sti.eduscol.education.fr//si-ens-paris-saclay/ressources_pedagogiques/caracterisation-mecanique-dun-tissu-mou-le-muscle-passif.

[10] SAUTY Bastien LAVIGNE Thomas. *Introduction à la Biomécanique - CultureSciences de l’Ingénieur*. https://sti.eduscol.education.fr//si-ens-paris-saclay/ressources_pedagogiques/introduction-a-la-biomecanique.

[11] Arkady Rutkovskiy Anna Malashicheva Gareth Sullivan Maria Bogdanova Anna Kostareva Kåre-Olav Stensløkken Arnt Fiane Jarle Vaage. “Valve Interstitial Cells: The Key to Understanding the Pathophysiology of Heart Valve Calcification”. In: *Journal of the American Heart Association* (2017). DOI: 10.1161/JAHA.117.006339.

[12] K. Yassuda. “Investigation of the analogies between viscometric and linear viscoelastic properties of polystyrene fluids”. PhD thesis. Massachusetts Institute of Technology, 1979.

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